Overlooked Market Risk Shocks: Prepayment Uncertainty and Option-Adjusted Spreads

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Assessments of market risk for economic or regulatory capital typically involve calculating a portfolio’s sensitivity to key risk-factor movements. Historically, practitioners in fixed-income markets have focused on two classical sources of risk, adverse changes in interest rates and volatility. As stress testing has evolved, researchers have identified additional market risk factors that can affect option-adjusted spreads (OAS) and the value of mortgage securities (Kopprasch [1994]; Heidari and Wu [2004]; Levin and Davidson [2005]). We propose including shocks to either mortgage security prepayment rates or option-adjusted spreads, which historically have been calibrated to accompany a corresponding set of interest rate and volatility shocks.

When stress testing, unanticipated changes in prepayment speeds can be accounted for by including model error adjustments. With access to a sufficient amount of historical model performance data, prepayment model error can be accurately measured and the resulting shocks can be well defined. However, this type of in-sample adjustment is backward-looking and fails to account for potential model advancements, forecasting error, and structural shifts in the consumer lending market (Fabozzi [1999]; Hayre and Rajan [1995]). In contrast, OAS serve as a broader and forward-looking measure of model error, which can encompass prepayment model errors due to misspecification as well as errors in forecasting (Babbel and Zenios [1992]). These shocks can be constructed based upon multiple historical vendor quotes, but their principal components are model dependent and vary across market environments (Heidari and Wu [2004]). Data scarcity is also a concern for non-agency securities.

To fully account for the risks associated with embedded optionality, we propose to include either prepayment error adjustments or OAS shocks when stress testing. Each type of shock has a distinct set of advantages as well as potential concerns. To generate prepayment model error adjustments and OAS shocks, which are consistent with simultaneous movements in the term structure of interest rates and implied volatility, we employed an adaptation of the linkage methodology developed by Bogin and Doerner [2014]. In this earlier work, the authors used historical reduced-form dependencies to model the relationship between interest rates and implied volatilities. We have used this same manner of linkage to associate interest rate movements to prepayment model error and OAS. Generating an internally consistent set of risk-factor movements allows for a more accurate measure of market risk, which can be used to inform institutions about the amount of capital needed to withstand a combination of adverse market events (Berkowitz, [1999]). Furthermore,
TWO METHODS FOR EQUILIBRATING MODEL AND MARKET PRICE

Prepayment models generate projected mortgage security cash flows that are then discounted to solve for the associated security’s model price. When these model prices diverge from market prices, there are two potential corrections to impose alignment—conditional prepayment rate (CPR) adjustments and OAS adjustments. Historically, CPR adjustments have been used in modeling credit risk (which is primarily concerned with cash flows) while OAS adjustments have been used in modeling market risk (which is primarily concerned with valuation). The two ideas, though, have mathematical overlaps, and connecting them could lead to better risk-management practices. Consider a standard loan pricing formula,

\[ P = \sum_{k=1}^{360} \frac{C_k}{\left(1 + \frac{r_k}{12} + \theta\right)^k} \]

where \(\{C_k\}, k = 1, 2, 3, \ldots, 360\) are 360 months of expected principal and interest cash flows. These cash flows are discounted using a vector of spot rates \(\{r_k\}, k = 1, 2, 3, \ldots, 360\) to calculate the present value of the mortgage security. If the mortgage security’s model price differs from its market price, we can either adjust prepayment speeds, which affect cash flows, \(C_k\), or we can add a constant spread, \(\theta\) (OAS), to the discount rate in the denominator to equilibrate market and model prices.5 Note, a divergence between market quotes and the associated security’s model price, when these model cash flows are discounted using a vector of spot rates to calculate the present value of the associated mortgage security. One of the primary risks in projecting cash flows comes from prepayment model error, which can arise from two sources—macroeconomic forecasts that deviate from scenarios actually observed, or out-of-model error, and model misspecification, or in-model error (Borodovsky and Lore [2000]). Because we planned to couple our prepayment shocks with simultaneous movements in the term structure of interest rates and implied volatility, which already shift the underlying macroeconomic environment, we focused our attention on in-model error.

To isolate in-model error, we generated model predictions using actual or historically accurate inputs. We then compared these model predictions with realized conditional prepayment rates. Any deviation of predicted versus realized CPR is due to model misspecification and can arise for several reasons. First, prepayment realizations are subject to random shocks and therefore model projections will contain purely statistical error, even for a fixed economic scenario. Second, as market conditions change, prepayment model performance can deteriorate due to misspecified or omitted dependencies among the included model variables. Third, in-model error can persist even when the model is re-parameterized, particularly if there is insufficient information to determine whether structural shifts are temporary or permanent (Hayre and Rajan [1995]; Burns [2010]).

After calculating the magnitude of in-model error across several prepayment models and mortgage securities, we generated an initial set of prepayment model error shocks. We then adjusted these based on scenario-specific interest rate changes and portfolio specific loan and prepayment model characteristics. The resulting scenario-specific prepayment model error adjustments are well defined, but potentially too narrow in scope to capture the full extent of prepayment error (Hayre and Rajan [1995]). Further, these sets of historically based shocks are backward-looking and fail to account for potential model advancements or structural shifts in the consumer lending market (Fabozzi [1999]).

PREPAYMENT RATES

Prepayment models are deployed in mortgage security valuation to estimate pool or loan-level cash flows conditional on a set of collateral characteristics and a macroeconomic forecast.6 These cash flows are then discounted using a projected interest rate path to solve for the present value of the associated mortgage security. The two ideas, though, have mathematical overlaps, and connecting them could lead to better risk-management practices. Consider a standard loan pricing formula,

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Methodology

Our methodology for constructing prepayment model error adjustments is predicated on two overarching design goals: generalizability and ease of
implementation. After explaining each goal below, we present summary statistics along with visual examples to describe the data and model predictions.

The generalizability of our prepayment model error adjustments is largely a function of the quality and quantity of available data. In an effort to incorporate prepayment model performance across a wide array of loan types, we used historical performance data from several leading vendors. These data provide model-predicted and historically observed single monthly mortalities (SMM) across different market environments. Unfortunately, many of the included time series are brief in nature (e.g., model performance of loans originated with a 3% coupon are only available for two years), which prevented us from constructing a robust measure of model error across origination year, coupon, or loan type. Instead, we constructed a general prepayment model error adjustment that represents model performance across an array of loans.

The historical performance data allowed us to compute in-model error by annualizing monthly mortality rates as $\text{CPR} = 1 - (1 - \text{SMM})^{1/12}$ using both model-predicted and historically observed SMM. We then took the ratio of these CPR measures to calculate a CPR multiplier to correct for over- and under-prediction of prepayment speeds: $\text{CPR multiplier} = \frac{\text{CPR}_{\text{realized}}}{\text{CPR}_{\text{predicted}}}$. We chose this type of adjustment measure because it can be broadly applied across the majority of commercially available valuation models that include a tool to speed up or slow down prepayment rates using a multiplier on predicted CPR. If the multiplier is above one, the model predicted CPR speeds are too slow vis-à-vis observed prepayment speeds and require speeding up. Alternatively, if the multiplier is below one, predicted CPR speeds are too fast vis-à-vis observed prepayment speeds and require slowing down. In this respect, the extent to which predicted prepayment speeds are too fast or too slow is the reciprocal of our CPR multiplier, with the multiplier itself representing the correction. In the following subsections, we analyze errors associated with over-prediction and under-prediction separately to allow for asymmetry in the error distributions. This leads to two sets of CPR multipliers to capture over- and under-estimates of actual prepayment speeds.

Datasets

To estimate the extent of in-model error, we employed historical performance data from several agency and non-agency prepayment models from 2000 to 2013. The data include detailed information on realized and predicted CPRs for a variety of loan types. Finding that across vendors, prepayment models have qualitatively similar in-model errors, we aggregated results across vendors and in this article present an overall assessment. CPR multipliers are detailed in Exhibit 1 with multipliers above one presented in panel (A) and multipliers below one presented in panel (B). These statistics are described in greater detail throughout the next several subsections.

### Agency Model Prepayment Multiplier

The historical performance data for agency prepayment models include realized and predicted CPRs for several mortgage types. The data contain 55,216 observations, which represent an unbalanced panel of 563 coupon/origination year combinations. For the fixed-rate mortgages, the bulk of the data are concentrated in 30- and 15-year maturities, which contain 73.9% of available observations among fixed-rate loans.

### Exhibit 1

<table>
<thead>
<tr>
<th>Type</th>
<th>Average Value Range</th>
<th>Median</th>
<th>Average Value Range</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(A) Multipliers Above One</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed Rate</td>
<td>1.25</td>
<td>1.16–1.46</td>
<td>1.30</td>
<td>1.15–1.58</td>
</tr>
<tr>
<td>Hybrid and ARM</td>
<td>1.30</td>
<td>1.20–1.54</td>
<td>1.44</td>
<td>1.40–1.46</td>
</tr>
<tr>
<td>Agency Securities Average</td>
<td>1.45</td>
<td>1.10–1.16</td>
<td>1.14</td>
<td>1.10–1.16</td>
</tr>
<tr>
<td>Alt-A Fixed, Hybrid, and ARM</td>
<td>1.30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jumbo Fixed and Hybrid</td>
<td>0.85</td>
<td>0.80–0.89</td>
<td>0.70</td>
<td>0.54–0.82</td>
</tr>
<tr>
<td>Subprime Fixed and Hybrid</td>
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<td>0.32–0.86</td>
<td>0.70</td>
<td>0.54–0.82</td>
</tr>
<tr>
<td>Non-Agency Securities Average</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(B) Multipliers Below One</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed Rate</td>
<td>0.85</td>
<td>0.80–0.89</td>
<td>0.73</td>
<td>0.32–0.86</td>
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<tr>
<td>Hybrid and ARM</td>
<td>0.70</td>
<td>0.55–0.85</td>
<td>0.70</td>
<td>0.54–0.82</td>
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<tr>
<td>Agency Securities Average</td>
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<tr>
<td>Alt-A Fixed, Hybrid, and ARM</td>
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<tr>
<td>Jumbo Fixed and Hybrid</td>
<td>0.70</td>
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<tr>
<td>Non-Agency Securities Average</td>
<td>0.70</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Model errors are shown for agency and non-agency securities grouped by mortgage type and conditional on multipliers above or below one. The multipliers are defined as the ratio of the model-implied CPR to the realized CPR.
For each loan type combination, the following variables are available: date, model SMM, historical SMM, origination year, product type, and a coupon, if applicable.

We begin our analysis of in-model prepayment error by examining the distribution of CPR multipliers across fixed-rate mortgages (FRMs). Multipliers above one represent under-estimates of actual prepayment speeds and range from 1.16 for 40-year fixed-rate mortgages to 1.46 for 10-year FRMs. The average median multiplier above one is 1.25. Multipliers below one represent over-estimates and their medians range from 0.80 for 20-year fixed-rate mortgages to 0.89 for 10-year fixed-rate mortgages. The average median multiplier below one is 0.85. Among all fixed-rate mortgage types, multipliers above one occur more frequently than multipliers below one.

Next, we turn to data for hybrids and adjustable-rate mortgages (ARMs). The upper bound for hybrid mortgage median multipliers associated with under-prediction is considerably higher than that for fixed-rate mortgages—1.58 for GNMA 10-year resets. We also observe a higher upper bound for ARMs. The average median multiplier across hybrids and ARMs is 1.30. For over-predictions, median multipliers range from 0.32 to 0.89 with an average of 0.73, which is slightly below the FRM average. Given the similarity of median multipliers across both product type and securitizing agency, prepayment rate error can be broadly captured with only two sets of multipliers: one for the impact of over-estimating prepayment speeds and another for the impact of under-estimating prepayment speeds.11

Non-Agency Model Prepayment Multiplier

The historical performance data for non-agency models cover 10 types of loans: Alternative A-paper (Alt-A) 15- and 30-year fixed-rate mortgages, Alt-A 5-year hybrid and 30-year option ARMs, jumbo 15- and 30-year fixed-rate and 5-year hybrid ARMs, and subprime fixed-rate 30-year and 2- and 3-year hybrid ARMs. The data contain 10,236 monthly observations that include information on type of mortgage, origination year, model SMM, and historical SMM. The observations are evenly apportioned across loan types; however, some type/origination year combinations exit the data prior to 2013.

Turning to the CPR multipliers, for model errors above one (under-estimates), the smallest non-agency median multiplier is equal to 1.10 and is associated with subprime three-year hybrid mortgages. The largest non-agency median multiplier is equal to 1.54 and is associated with Alt-A fixed-rate 15-year mortgages. Alt-A mortgages, excluding option ARMs, have an average median multiplier of 1.45. In contrast, all subprime and Alt-A option ARMs have median multipliers below 1.20. Across all mortgage types, the average median multiplier is 1.33. For model errors below one (over-estimates), median multipliers range from 0.54 for jumbo 15-year fixed-rate mortgages to 0.85 for Alt-A 30-year fixed-rate mortgages. The average median across all loan types is 0.68.

In contrast to the agency model results, non-agency multipliers below one occur more frequently than multipliers above one. Non-agency models tend to over-predict prepayment speeds while agency models under-predict them. This finding is consistent with anecdotal evidence from the last crisis when borrowers with lower credit quality represented a disproportionate share of non-agency loans and were often unable to refinance at lower interest rates.

Interest Rates and Changes in the CPR Multiplier

The analysis in the prior subsections suggests that prepayment model errors are relatively consistent across vendors, issuers, and mortgage types. Specifically, we find that median CPR multipliers representing over-prediction are clustered around 0.8 and median CPR multipliers representing under-prediction around 1.3. These estimates are similar in size to the ±20% uncertainty band that Fabozzi [1999] suggests practitioners should use to account for prepayment model error.12 The top panel of Exhibit 2 depicts the relationship between changes in an empirical measure of the Libor-Swap curve’s slope (10-year Swap rate minus three-month Libor rate) and changes in prepayment model error.13 As illustrated, the degree of prepayment error is sensitive to curve steepening. We have accounted for this, as well as other interrelationships, by calculating add-ons to the two median multipliers that are consistent with simultaneous movements in the term structure of interest rates and implied volatility.

Each set of add-ons has been estimated using historical loan performance data from the agency prepayment models and daily trading data from Bloomberg. Using the Bloomberg data, we created a time series of historical...
Libor-Swap and Agency par yields. We converted the Libor-Swap and Agency par yields into spot rates using bootstrapping. Then, we parameterized these spot trades using the Björk-Christensen [1999] factorization, which allowed us to describe the historical term structure of rates using only five parameters:

\[
\gamma(t) = \beta_1 + \beta_2 \left( \frac{t}{2} + \frac{1 - e^{-\lambda t}}{\lambda t} \right) + \beta_3 \left( \frac{1 - e^{-\lambda t}}{\lambda^2 t} - \frac{e^{-\lambda t}}{\lambda} \right) + \beta_4 \left( \frac{1 - e^{-2\lambda t}}{2\lambda t} \right) + \beta_5 \left( \frac{1 - e^{-2\lambda t}}{\lambda^2 t} \right)
\]

(2)

where \(\gamma(t)\) is the yield at term point \(t\), and \(\lambda\) is an exponential decay factor. Similar to Bogin and Doerner [2014], we model simultaneous six-month changes in interest rates and in-model error as

\[
\Delta \text{CPR} = \alpha_i + \sum_{j=1}^{3} \phi_j \text{CPR} \text{Mul}_{i-H,j} + \sum_{j=1}^{5} \eta_j \text{Ag}_{i-H,j} + \sum_{j=1}^{5} \xi_j \Delta \text{Beta}_{i,j} \text{(Libor-Swap)} + \sum_{j=1}^{5} \zeta_j \Delta \text{Beta}_{i,j} \text{(Agency)} + \varepsilon_{i,j}
\]

(3)

Note: In panels (B) and (C), the time series show predicted and actual six-month changes in the CPR multiplier. The CPR multiplier regression model, as expressed in Equation (2), is used to compute scenario-specific prepayment model error adjustments. Actual data are based on 30-year fixed-rate mortgage for Fannie Mae (FNMA) and Freddie Mac (FHLMC).
where $\Delta CPR\ Multi_{i,j}$ is the six-month change in the CPR multiplier (realized/predicted) associated with loan $j$ between period $i$ and $i-H$. $Age_{i-H,j}$ measures the number of months since origination, and the final two terms capture the effect of contemporaneous changes in the Libor-Swap and Agency interest rate curves. Equation (3) yields adjusted $R^2$ values that range from 0.43 to 0.45. This strength is notable given that the dependent variable is effectively a credit model error term and represents unexplained variation in prepayment model error. Strength of fit is further evidenced in the bottom two panels of Exhibit 2, which depict actual versus predicted changes in the CPR multipliers for 30-year fixed-rate FNMA and FHLMC mortgages. Following the 2001 and 2007–2009 recessions, we saw slightly degraded unadjusted model fit because of market uncertainty and unprecedented government interventions. The add-ons helped correct for such model error as shown by the how well the predictions track the actual series in both panels.

To calculate the add-on associated with each of our interest rate scenarios, we used Equation (3). $CPR\ Multi_{i-H,j}$ is first set equal to 1.3, the median CPR multiplier associated with under-prediction. For purposes of exposition, we drew changes in the refinance incentive and loan age from the last three vintages prior to the analysis date. For each vintage, we calculated a predicted six-month change in the associated CPR multiplier, which we then averaged across these predictions to arrive at an add-on, or adjustment to the 1.3 median CPR multiplier for each scenario. We repeated this process for the median multiplier associated with over-prediction, 0.8.

To ensure a sufficient amount of stress, we constrained the under-prediction add-on to be greater than or equal to zero (resulting in a minimum scenario-specific multiplier of 1.3) and constrained the over-prediction add-on to be less than or equal to zero (resulting in a maximum scenario-specific multiplier of 0.8). The resulting scenario-specific CPR multipliers are illustrated in Exhibit 3. As shown, the adjustments are largely dominated by the initial shock to prepayment speeds associated with each median multiplier. Most of the deviations came in the post-recessionary periods after the dot-com bubble and Great Recession. Since prepayment models do not account for credit and liquidity risks—both issues during those times—we turn to OAS shocks.

**OPTION-ADJUSTED SPREADS**

OAS is the constant spread added to the discount rates used to calculate the present value of a mortgage security’s projected cash flows. It is used to equilibrate market quotes and model derived prices and encompasses model misspecification, forecasting errors, and liquidity risk and credit risk, where applicable (Levin...
and Davidson [2005, 2008]). OAS serves as a broad measure of model error; however, its primary components are model dependent and vary across market environments (Heidari and Wu [2004]). For any given mortgage security, OAS is constructed to equate the security’s model price to “fair value” market quotes. It encompasses prepayment model errors due to misspecification, as well as errors in forecasting the factors influencing cash flows and their associated discount rates (Babbel and Zenios [1992]).

From a practitioner’s viewpoint, OAS is often considered synonymous with the premium investors require for taking on prepayment risk. If borrowers were not permitted to prepay their mortgages or if the prepayment penalties were sufficiently onerous to eliminate the uncertainty associated with prepayments, OAS would be significantly muted. However, that is not the case; borrowers in the fixed-rate and adjustable-rate prime space generally incur no penalty fee for prepaying their mortgages, unlike in the subprime space, and may do so at any time.

For fixed-rate loan holders, if interest rates fall, borrowers are predisposed to exercise their options to prepay and refinance their mortgages at lower rates, in direct opposition to the best interest of investors, who would prefer to continue receiving the higher rate. Alternatively, if interest rates rise, borrowers have negligible incentive to prepay their mortgages (short of relocation or other turnover); investors, on the other hand, would prefer to see the below-market interest rate or discount loans disappear in favor of reinvesting at higher interest rates. The OAS required by investors for having given up the prepayment option depends in large part, then, on the accuracy of modeled prepayment speeds. The empirical challenge is to connect OAS with interest rates.

Option-Adjusted Spreads and Interest Rates

Changes in several types of macroeconomic factors, including the term structure of interest rates, can shift market expectations and mortgage security prices before they are fully incorporated into prepayment models through recalibration or tuning parameter adjustments (Kopprasch [1994]). These unexpected changes can lead to an increase in OAS, which decreases the value of the associated mortgage assets.

There is a strong empirical relationship between a mortgage security’s OAS and interest rates (Heidari and Wu [2004]). All else constant, a mortgage security’s OAS will widen when interest rates fall because when investors face a greater probability of borrowers exercising their prepayment options, they require a higher premium for taking the prepayment risk. Conversely, OAS will tighten when interest rates rise because when investors face a lower probability of borrowers exercising their prepayment options, a smaller premium is required for taking the prepayment risk (Green and Shoven [1983]). This response can also be thought of in mechanical terms. Assume a mortgage model’s pricing error remains fixed at $\theta$ across interest rate environments. As rates increase, mortgage prepayments slow and Macaulay duration, or the security’s weighted average term to maturity, increases. A narrower spread is required to eliminate $\theta$ because a given OAS has a greater effect on the present value of distant cash flows. Alternatively, as interest rates decrease, prepayments accelerate and Macaulay duration decreases. This leads to fewer distant cash flows, and a wider spread required to eliminate $\theta$ (Kupiec and Kah [1999]).

To link changes in OAS to simultaneous movements in interest rates, we have modeled their reduced-form historical relationship by leveraging the methodology proposed by Bogin and Doerner [2014]. By calculating OAS shocks conditional on contemporaneous changes in the term structure of interest rates, we have ensured an internally consistent set of risk-factor movements. Because OAS is sensitive to the choice of both term structure and prepayment model, we worked with time series data supplied by multiple vendors (Babbel and Zenios [1992]) to mute the effect of individual idiosyncrasies.

Each OAS time series tracks the spread attached to current coupon to-be-announced (TBA) agency issues. We calculated six-month historical changes in OAS to serve as the dependent variable in our model linking interest rates to OAS. This allowed us to project changes in OAS predicated on a set of simultaneous interest rate shocks. By including these shocks when stress testing, we allowed baseline OAS to change along with interest rates during portfolio revaluation.

Exhibit 4 presents three graphics that show historical OAS time series, regression estimation results, and scenario-specific shocks. In panel (A), the figure illustrates FNMA current coupon OAS based on several vendor prepayment models. As shown, FNMA OAS has historically ranged from –22 bps to 106 bps.
Even with such variation, we were still able to predict model changes in agency OAS with decent accuracy. To model the historical relationship between changes in interest rates and changes in OAS, we used the following reduced form specification:

$$\Delta OAS_i = \alpha_0 + \varphi OAS_{i-H} + \sum_{k=1}^{5} \gamma_k \Delta \beta_{k,i} (\text{Agency}) + \sum_{k=1}^{5} \delta_k \Delta \beta_{k,i} (\text{Libor-Swap}) + \varepsilon_i$$

(4)

This model is able to explain the majority of variation in historical six-month changes in current coupon OAS supplied by all vendors.\textsuperscript{20,21} The model fit is high, with adjusted $R^2$ values ranging from 0.63 to 0.75, as shown in panel (B) of Exhibit 4, for current coupon FNMA TBA issuances. The OAS model fit is stronger than that obtained with prepayment models in Exhibit 2.

After estimating Equation (4), it was straightforward to calculate the changes in current coupon OAS most likely to accompany a set of Libor-Swap and Agency interest rate shocks. To ensure a sufficient level of stress, we floored scenario-specific OAS shocks at zero. Panel

**EXHIBIT 4**

**OAS Time Series, Estimations, and Scenario-Specific Shocks**

**A** Historical OAS Time Series

**B** Estimated 6-Month Changes in Agency OAS

**C** Scenario-Specific OAS Shocks

*Note: In panel (A), the historical OAS time series shows daily OAS for the FNMA current coupon OAS (FNCL 30-Year CC OAS), which was calculated based upon the average of several vendor quotes. Panel (B) shows the OAS regression model used to compute scenario-specific OAS shocks for a single vendor time series. The six-month changes were computed from daily OAS (FNCL 30-Year CC OAS). In panel (C), each OAS shock was calculated as the median of vendor-specific model projections.*
(C) of Exhibit 4 illustrates estimated changes in current coupon OAS given varying sets of interest rate shocks. The simulated OAS shocks range in value from 0 bps to 28 bps with an average change of 8 bps, which is largely in line with the 10 bps model risk add-on calculated by Heidari and Wu [2004]. Hence, as changes to the interest rate environment begin to affect modeled prepayment speeds, we have established a way to generate, link, and select appropriate OAS shocks (i.e., the largest modeled projections in panel (C) might appeal to the most conservative investment strategies).

CONCLUSION

This article describes and shows the development of methodologies for generating shocks to prepayment rates and mortgage security OAS. Changes in prepayment speeds or a market risk factor that impacts mortgage security OAS can have a pronounced effect on both the asset and equity valuation of institutions holding large portfolios of mortgage securities.

We calculated our prepayment rate shocks using a monthly CPR multiplier, which we computed as the ratio of realized to predicted prepayment speeds. Since the distribution of in-model errors is different for under- and over-predictions, we have proposed an asymmetric prepayment error adjustment. For errors above one (under-estimates), we recommend multiplying model predicted CPR by a factor of 1.30. For errors below one (over-estimates), we recommend multiplying model predicted CPR by a factor of 0.80. To fully capture the extent of potential in-model risk, both multipliers (1.30 and 0.80) should be incorporated in stress testing. After calculating a robust set of median CPR multipliers, we developed scenario-specific add-ons to ensure our prepayment model error adjustments were consistent with simultaneous interest rate and implied volatility shocks. The resulting scenario-specific prepayment model error adjustments are well defined but potentially too narrow in scope to capture the full extent of prepayment error.

Mortgage security OAS serves as a potentially broader measure of model error, which encompasses both misspecification and errors in forecasting model inputs. Our OAS shocks are based upon historical six-month changes in spreads and calculated using data derived from competing prepayment models. To ensure consistency with shocks to other key risk factors, we modeled the relationship between six-month changes in spreads and contemporaneous changes in the term structure of interest rates. Regression results can be used to generate anticipated OAS shocks given a set of simultaneous and scenario-specific interest rate changes.22

We have proposed including changes to either prepayment rates or OAS when stress testing to more accurately measure market risk. We combine data from different models to arrive at a single set of shocks. For practical implementation, this type of aggregation is desirable but results in a loss of information regarding the distribution of shocks calculated using competing models. Since this article provides a series of steps to show how these shocks can be constructed, we hope the applied quantitative approach will enhance portfolio management practices. Quite often, such information is part of proprietary techniques that are used for private investment gains. Smaller companies may not have the staff or resources to perform this kind of portfolio research. Future studies might analyze the tradeoffs between OAS and prepayment rate shocks by applying each set of shocks to a representative portfolio of mortgage assets. Each set of shocks could be applied to a representative portfolio of mortgage assets to examine the resulting capital implications under a range of models and market environments.

ENDNOTES

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1This is often accomplished through either Monte Carlo or historical simulations. Monte Carlo is sensitive to the choice of distributions from which risk factors are drawn. Instead, we focus in this article on historical simulations, which require few parametric assumptions.

2A current and best practice among regulators is to include shocks to option-adjusted spreads when market risk stress testing, particularly to shock portfolios containing fixed-income securities with embedded optionality (e.g., mortgage backed securities, callable debt). For instance, option-adjusted spreads are included as a risk factor in the Global Market Shock under the Federal Reserve’s Comprehensive Capital Analysis and Review (CCAR).

3Bogin and Doerner’s [2014] methodology represents a departure from traditional historical simulation. Under the traditional historical simulation approach, a set of historically based interest rate shocks drive variation in other, portfolio-relevant risk factors using a variant of the factor model. Because the relationship between interest rates and these other
risk factors is static, the methodology suffers from some of the same flaws as standard historical simulation (e.g., it is unable to account for time varying volatilities and changes in correlation across risk factors). More information on the potential flaws of historical simulation is discussed by Pritsker [2006]. An alternative approach to constructing shocks in low interest rate environments is outlined by Abdymomunov and Gerlach [2014].

Based on the analysis presented in this article, we recognize that prepayment model error and OAS are model-dependent. This presents several challenges for practical implementation. Despite these challenges, we believe that it is important to account for prepayment uncertainty when measuring market risk.

To compare these two approaches, we can compute both the OAS and the prepayment adjustment that would equilibrate model and market prices for a representative 30-year fixed-rate loan. As an example, let us assume the following loan characteristics: a 7% coupon rate, a model CPR of 15%, and a face value of $100,000. When the discount rate associated with this loan is 4% (with a zero OAS), the model price is equal to $114,372.29. If the market is pricing the loan at $111,891.37, one interpretation is that our prepayment model is too slow relative to the market. A tuning adjustment of the CPR by 1.3 (generating a CPR of 19.5%), would equilibrate market and model prices. Alternatively, we could include an OAS of 48 basis points (bps) to the discount rate to equilibrate market and model prices. In the former case, the projected timing of the cash flows is adjusted to be paid to the investor sooner, which lowers value. In the latter case, the cash flows remain the same, but they are discounted at a higher rate yielding a lower modeled value for the instrument.

Credit models usually differentiate between voluntary and involuntary prepayments (defaults), whereas standard prepayment models or total terminations models typically do not. Typically, a total termination prepayment model is used to measure market risk.

The single monthly mortality rate is the percentage of the principal amount of mortgages that are prepaid in a given month. It is used for tracking prepayments in a mortgage pool.

The prepayment rate errors associated with the prepayment models we examined are higher during the first three to five years of a mortgage’s life and lower thereafter.

We analyzed historical agency prepayment models performance data for the following types of loans: Federal National Mortgage Association (FNMA) and Federal Home Loan Mortgage Corporation (FHLMC) fixed-rate 30-, 20-, 15-, and 10-year mortgages; FNMA fixed-rate 40-year mortgages; Government National Mortgage Association (GNMA) fixed-rate 15- and 30-year mortgages; FNMA and FHLMC 10-, 7-, 5-, and 3-year hybrid mortgages; GNMA 5- and 3-year hybrid mortgages; and FNMA and FHLMC 30-year adjustable-rate mortgages.

In constructing group averages, the median multipliers associated with each loan type are equally weighted.

Institutions with large mortgage exposures may want a more disaggregated set of prepayment speed shocks.

This is a general measure of prepayment model error, which Fabozzi [1999] believes could be refined given an investor’s specific portfolio.

The adjusted $R^2$ of the associated regression is equal to 0.075. The slope coefficient, $\hat{\beta}$, is equal to 8.54 and statistically significant at the 1% level.

A detailed description of this factorization is presented in Björk and Christiansen [1999].

The data underlying Equation (3) satisfy the classical linear assumptions necessary to ensure an unbiased and consistent least squares estimator. Further, model specification tests suggest a lack of specification error. All included variables have been tested for unit roots.

This assumes full market efficiency, which does not hold in practice (Shleifer [2000]; Mason and Rosner [2007]).

If a mortgage security’s model price exceeds its market price, a positive OAS can be added to each monthly discount rate, leading to a lower model value. Alternatively, if the mortgage security’s model price is less than its market price, a negative OAS can be added to each monthly discount rate, leading to a higher model value.

Non-agency OAS also incorporates an investor premium for taking on varying degrees of liquidity and credit risk.

A negative OAS increases the present value of projected cash flows, which increases a mortgage security’s model price.

The data underlying Equation (4) satisfy the classical linear assumptions necessary to ensure an unbiased and consistent least squares estimator and model specification tests suggest a lack of specification error. All included variables have been tested for unit roots.

The $\Delta \hat{\beta}_{k,i}$ are based upon the Björk-Christensen [1999] parameterization described in the earlier section on interest rates and changes in the CPR multiplier.

While broad in scope, these shocks are based upon a set of specific vendor quotes that are model dependent.

REFERENCES


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